Find the **polar coordinates** of all intersection points of the polar curves $r = 1 + 2\cos 2\theta$ and $r = 1$. **SCORE: _____ / 8 PTS**
You must find your answers algebraically, **NOT by graphing, trial & error or guess & check**.

$1 + 2\cos 2\theta = 1 \qquad \theta \in [0, 2\pi]$

1 $\cos 2\theta = 0, \qquad 2\theta \in [0, 4\pi]$

$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$ 1 $\frac{1}{2}$ $\longrightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

3

$\frac{1}{2}$ POINT EACH

$\left(1, \frac{\pi}{4}\right)$

$\left(1, \frac{3\pi}{4}\right)$

$\left(1, \frac{5\pi}{4}\right)$

$\left(1, \frac{7\pi}{4}\right)$

$\left(-1, \frac{\pi}{2}\right)$ OR $\left(1, \frac{3\pi}{2}\right)$

$\left(-1, \frac{3\pi}{2}\right)$ OR $\left(1, \frac{\pi}{2}\right)$

$-r = 1 + 2\cos 2(\pi + \theta) \qquad -r = 1$

$-r = 1 + 2\cos 2\theta \qquad r = -1$

$r = -1 - 2\cos 2\theta$

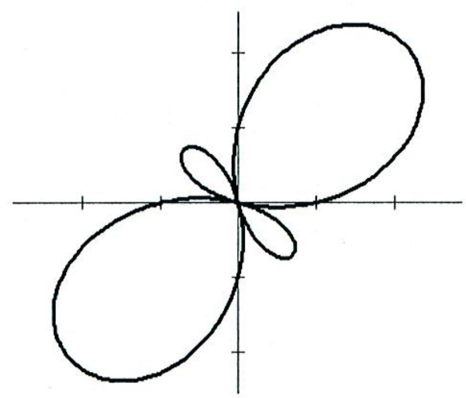
1 $1 + 2\cos 2\theta = -1,$ (EQUIVALENT TO $-1 - 2\cos 2\theta = 1$)

$\frac{1}{2}$ $\cos 2\theta = -1$

1 $2\theta = \pi, 3\pi \longrightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$-1 - 2\cos 2\theta = -1$ EQUIVALENT TO $1 + 2\cos 2\theta = 1$

The graph of the polar curve $r = 1 + 2\sin 2\theta$ is shown on the right. **SCORE: _____ / 8 PTS**



[a] Find the slope of the tangent line to the curve at the $y-$intercept above the origin.
NOTE: You may find the slope without completely simplifying the derivative.

$$\frac{dy}{dx} = \boxed{\frac{4\cos 2\theta \sin\theta + (1 + 2\sin 2\theta)\cos\theta}{4\cos 2\theta \cos\theta - (1 + 2\sin 2\theta)\sin\theta}} \quad 2$$

$$\left.\frac{dy}{dx}\right|_{\substack{\theta = \frac{\pi}{2} \\ r = 1}} \ \tfrac{1}{2} = \frac{4(-1)(1) + 0}{0 - (1)(1)} = 4 \quad 1$$

[b] Write, **BUT DO NOT EVALUATE**, one integral for the area inside one of the **smaller** loops.
You must find the limits of integration algebraically.

1 $1 + 2\sin 2\theta = 0, \qquad \theta \in [0, 2\pi]$

$\sin 2\theta = -\frac{1}{2} \qquad 2\theta \in [0, 4\pi]$

$2\theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6} \longrightarrow \theta = \underbrace{\frac{7\pi}{12}, \frac{11\pi}{12}}_{Q_2}, \underbrace{\frac{19\pi}{12}, \frac{23\pi}{12}}_{Q_4}$

GRAPH IN Q_4 — GRAPH IN Q_2

$$\frac{1}{2} \int_{\frac{7\pi}{12}}^{\frac{11\pi}{12}} (1 + 2\sin 2\theta)^2 \, d\theta$$

$\frac{1}{2}$ 2 1

Let P be the point $(-3, 2, -7)$. **SCORE: _____ / 6 PTS**

[a] **FILL IN THE BLANKS** using "to the left of", "to the right of", "in front of", "behind", "above" or "below":

Using the standard axes shown in lecture, P is BELOW the xy – plane,

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TO THE RIGHT OF the xz – plane, and

BEHIND the yz – plane.

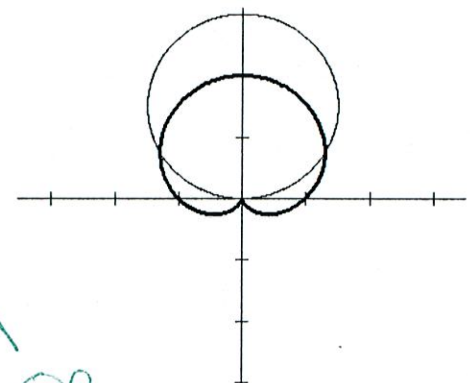
[b] The distance from P to the yz – plane is 3.

[c] The equation of the plane parallel to the xz – plane that passes through P is $y=2$.

[d] The equation of the sphere with center P that touches (is tangent to) the xy – plane is

$(x+3)^2+(y-2)^2+(z+7)^2=49$.

The graphs of the polar curves $r = 3\sin\theta$ (the circle) and $r = 1+\sin\theta$ (the cardioid) are shown on the right. **SCORE: _____ / 8 PTS**
Find the area of the region inside the circle and outside the cardioid.
You must find the limits of integration algebraically.



$$3\sin\theta = 1+\sin\theta$$
$$\sin\theta = \tfrac{1}{2}$$
$$\theta = \tfrac{\pi}{6}, \tfrac{5\pi}{6}$$

1 $\quad$ $1\tfrac{1}{2}$

$$\tfrac{1}{2}\quad \boxed{\tfrac{1}{2}}\boxed{\int_{\pi/6}^{5\pi/6}}\left((3\sin\theta)^2-(1+\sin\theta)^2\right)d\theta$$

OR

$$\int_{\pi/6}^{\pi/2}\left((3\sin\theta)^2-(1+\sin\theta)^2\right)d\theta$$

$$= \tfrac{1}{2}\int_{\pi/6}^{5\pi/6}\left(8\sin^2\theta-2\sin\theta-1\right)d\theta$$

OK IF SIMPIFIED $\quad$ $1\tfrac{1}{2}$

$$= \tfrac{1}{2}\int_{\pi/6}^{5\pi/6}\left(4(1-\cos 2\theta)-2\sin\theta-1\right)d\theta$$

$$= \tfrac{1}{2}\left(3\theta-2\sin 2\theta+2\cos\theta\right)\Big|_{\pi/6}^{5\pi/6}$$

2

$$= \tfrac{1}{2}\left(3\left(\tfrac{5\pi}{6}-\tfrac{\pi}{6}\right)-2\left(-\tfrac{\sqrt{3}}{2}-\tfrac{\sqrt{3}}{2}\right)+2\left(-\tfrac{\sqrt{3}}{2}-\tfrac{\sqrt{3}}{2}\right)\right)$$

$$= \tfrac{3}{2}\cdot\tfrac{2\pi}{3} = \pi \quad 1$$

BONUS: Write, **BUT DO NOT EVALUATE**, an integral (or sum or difference of integrals) for the area of the region inside the cardioid and outside the circle.

$\tfrac{1}{2}$ $\quad$ 1 $\quad$ 1

$$\tfrac{1}{2}\ \int_{5\pi/6}^{\pi}\left((1+\sin\theta)^2-(3\sin\theta)^2\right)d\theta + \int_{\pi}^{3\pi/2}(1+\sin\theta)^2\,d\theta$$